Study of the Radiation Reaction Force for a Step Electirc Field and an Electromagnetic Pulse

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In addition to Hammond method not presenting the so-called constant force paradox, using step force brings out the apparent physical contradictions of Landau-Lifshitz equation regarding energy conservation.
Unlike other cases, the electromagnetic pulse shows another fundamental difference between the two models.
Finally, an analysis of the Hammond method is made.

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Introduction

In 1938, Dirac [1] proposed a relativistic equation which includes the radiation reaction force for a spinless point-like charged particle.

Being a third-order differential equation, it do present solutions with physical anomalies such as self-accelerations and pre-accelerations.

In recent years, the Landau-Lifshitz equation [LL] [2] has been considered by many authors as the best equation to describe the motion of a spinless point-like charged particle including the radiation reaction force within the framework of Classical Electrodynamics.

The LL is a second-order differential equation and it does not present solutions with physical anomalies such as self-accelerations and pre-accelerations that exist in Dirac's theory.

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Different Proposals [3] (see Fig. 1)

Equation of Motion	$\dot{v}^\mu = (e/mc)F^{\mu\sigma}v_\sigma + \mathcal{G}^\mu$
LAD	$\mathcal{G}^\mu = \tau_0\left(\ddot{v}^\mu + v^\mu\dot{v}_\sigma\dot{v}^\sigma/c^2\right)$
LL	$\mathcal{G}^\mu = \tau_0\left((e/mc)\dot{F}^{\mu\sigma}v_\sigma + (e/mc)^2(F^{\mu\gamma}F_\gamma{}^\phi v_\phi + F^{\nu\gamma}v_\gamma F_\nu{}^\phi v_\phi v^\mu)/c^2\right)$
FO	$\mathcal{G}^\mu = +(e\tau_0/mc)\left(\frac{d}{d\tau}(F^{\mu\sigma}v_\sigma) - v^\mu v_\gamma\frac{d}{d\tau}(F^{\gamma\nu}v_\nu)/c^2\right)$
MP	$\mathcal{G}^\mu = (e_1/c)F^{\mu\sigma}\dot{v}_\sigma + (2e^2/3m^2c^6)F^{\nu\sigma}\dot{v}_\nu v_\sigma v^\mu$
SW	$\mathcal{G}^n = -\tau_0\omega^2\gamma^4 v^n$
HL	$\mathcal{G}^n = -\tau_0\gamma^6\dot{v}^2v^n/c^2$
Y	$\mathcal{G}^\mu = \theta(\tau)\tau_0\left(\ddot{v}^\mu + \frac{v^\mu}{c^2}\dot{v}_\sigma\dot{v}^\sigma\right)$
H	$\mathcal{G}^\mu = \phi^\mu - v^\mu\dot{\phi}/c^2$

Figure 1: LAD=Lorentz-Dirac; LL=Landau-Lifshitz; FO=Ford-O'Connel; MP=Mo-Papas; SW=Steiger-Woods; HL=Hartemann-Luhman; Y=Yaghjian; H=Hammond

Purpose

Hammond [3] noticed that when the LL is considered to describe the motion of a charged particle submitted to a constant electric field, the radiation reaction force vanishes and the solution is identical to the one obtained by using the Lorentz equation.

This is called the constant force paradox [4] [5].

Consequently, he proposed another method to describe the motion of a charged particle.

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The Landau-Lifshitz Equation

The Landau-Lifshitz equation of motion for a charged point particle is [2]

$$ma^\mu = (q/c)F^{\mu\nu}w_\nu$$
$$+\tau_o\left[\tfrac{q}{c}\left(\tfrac{\partial F^{\mu\nu}}{\partial x^\alpha}w^\alpha w_\nu - (q/cm)F^{\mu\nu}F_{\alpha w}w^\alpha\right) + (q^2/c^4m)F^2w^\mu\right]. \quad (1)$$

And after some algebra by defining [5],

$$\Delta^{\mu\nu}(w) = g^{\mu\nu} - \frac{w^\mu w^\nu}{c^2}, \quad (2)$$

we obtain

$$ma^\mu = \frac{e}{c}F^{\mu\nu}w_\nu + m\tau_o\Delta^{\mu\nu}(w)\frac{e}{mc}\left[\frac{e}{mc}F_{\nu\alpha}F^{\alpha\beta}w_\beta + w^\rho w^\alpha\frac{\partial F_{\nu\alpha}}{\partial x^\rho}\right] \quad (3)$$

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The Constant Force Paradox

Consider the radiation reaction term for a constant electric field E,

$$\Delta^{\mu\nu}(w_\rho)\left[\frac{e}{mc}\left[F_{\nu\alpha}\right]\left[F^{\alpha\beta}\right]w_\beta + w^\rho w^\alpha \frac{\partial F_{\nu\alpha}}{\partial x^\alpha}\right] = \Delta^{\mu\nu}(w_\rho)\left[\frac{e}{mc}\left[F^{ext}_{\nu\alpha}\right]\left[F^{\alpha\beta}\right]w_\beta\right]. \quad (4)$$

Then,

$$\Delta^{\mu\nu}(w_\rho)\left[\frac{e}{mc}\left[F_{\nu\alpha}\right]\left[F^{\alpha\beta}\right]w_\beta + w^\rho w^\alpha \frac{\partial F_{\nu\alpha}}{\partial x^\alpha}\right] = \left(n^{\mu\nu} - \frac{w^\mu w^\nu}{c^2}\right) \times \left[\frac{e}{mc}\left[F_{\nu\alpha}\right]\left[F^{\alpha\beta}\right]w_\beta\right] = E^2 w^\mu \left(1 - \frac{c^2}{c^2}\right) = 0. \quad (5)$$

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The Constant Force Paradox

Therefore, for the constant electric force, the LL is equivalent to the Lorentz equation of motion.

No radiation reaction force.

If we consider the electric field in the x^1 direction, the equations turn to be

$$\frac{dw^0}{d\tau} = \frac{eE}{mc} w^1 = \Omega w^1$$
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where $\Omega = \frac{eE}{mc}$. If we impose the initial conditions for the $4 - velocity$, $w^0 = c$ and $w^1 = 0$, the well-known solutions are

$$w^0 = c \cosh \Omega\tau$$
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Graph of w^1 vs τ for the Lorentz and the LL equations for $\Omega = 1$ with a Constant Electric Field

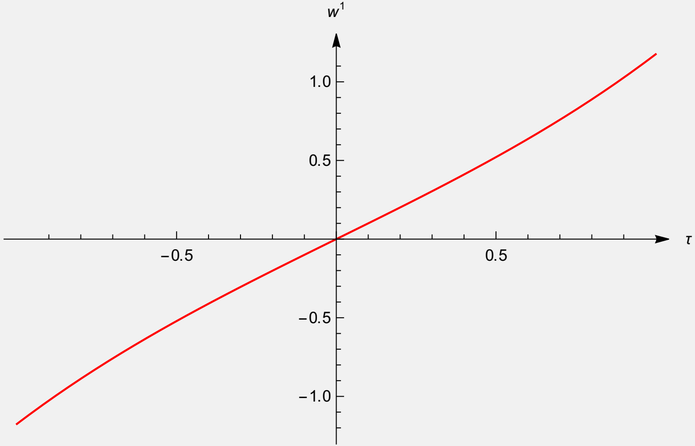

Figure 2: w^1 for the constant electric field in the x^1 direction by using the Lorentz and LL equations

Graph of w^0 vs τ for the Lorentz and LL Equations for $\Omega = 1$ with a Constant Electric Field

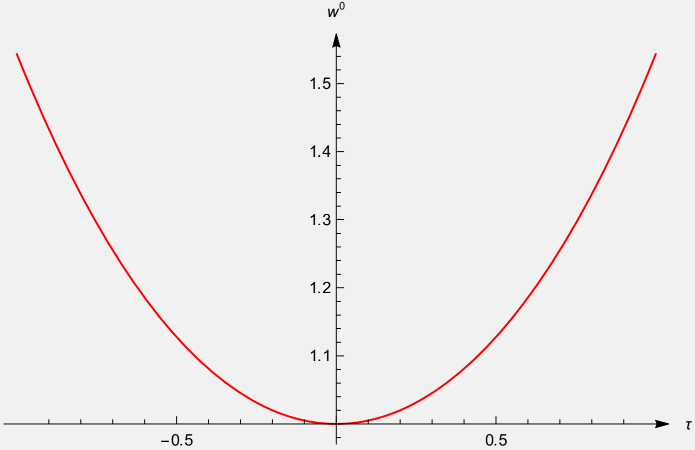

Figure 3: w^0 for the constant electric field in the x^1 direction by using the Lorentz and LL equations

Step Force for the Lorentz Equation

Let us consider an electric field in the x^1 direction which behaves as a step function; that is:

$$E = \left\{ \begin{array}{ccc} 0 & for & \tau < 0 \\ E_o & for & \tau \geq 0 \end{array} \right\} = E_o H(\tau) \tag{8}$$

The solutions for the Lorentz equation are simple:

1° case, $\tau < 0$.

The solution is

$$w^1 = 0 \qquad and \qquad w^0 = c \tag{9}$$

2° case, $\tau \geq 0$.

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$$w^1 = c \sinh \Omega\tau \qquad and \qquad w^0 = c \cosh \Omega\tau \tag{10}$$

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Graph of w^1 vs τ for the Lorentz Equation $\Omega = 1$ with a Step Electric Field

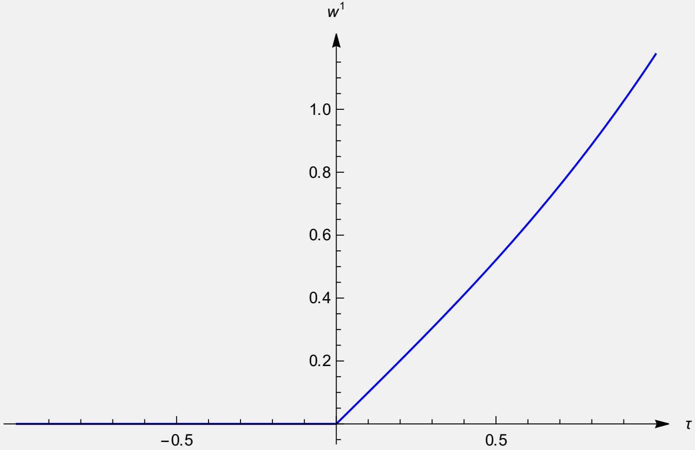

Figure 4: w^1 for the step electric field in the x^1 direction by using the Lorentz equation

Graph of w^0 vs τ for the Lorentz Equation $\Omega = 1$ with a Step Electric Field

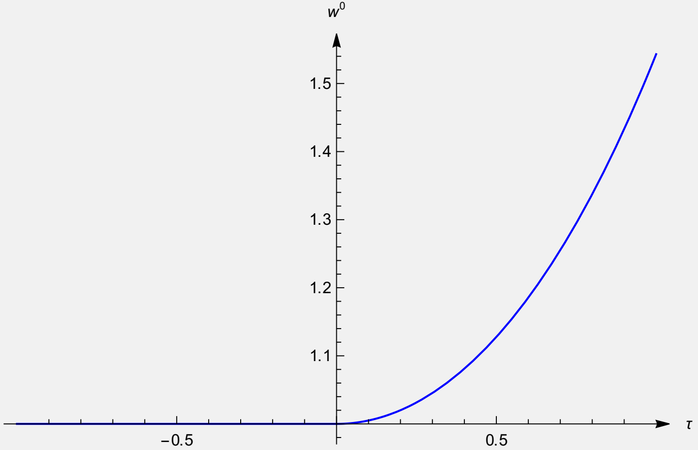

Figure 5: w^0 for the step electric field in the x^1 direction by using the Lorentz equation

Step Force for the LL

By using the LL, Eq. (3), for the step electric field, we have for w^1

$$\begin{aligned}\frac{dw^1}{d\tau} &= \Omega H(\tau) w^0 \\ &\quad +\tau_o \Omega \left[\delta(\tau) w^0 + H(\tau)\frac{dw^0}{d\tau}\right] \\ &\quad +\tau_o \Omega^2 H(\tau)^2 w^1,\end{aligned} \tag{11}$$

and for w^0,

$$\begin{aligned}\frac{dw^0}{d\tau} &= \Omega H(\tau) w^1 \\ &\quad +\tau_o \Omega \left[\delta(\tau) w^1 + H(\tau)\frac{dw^1}{d\tau}\right] \\ &\quad +\tau_o \Omega^2 H(\tau)^2 w^0.\end{aligned} \tag{12}$$

Step Force for the LL

These equations can be reduced in a simple fashion by using the fact that the LL reaction term vanishes with the constraint that at $\tau = 0$ the δ-function creates a jump and it turns out to consider the Lorentz equation just with different initial conditions due to the jump in each step.

Another way of solving the equation just consists in proposing a general solution of the type:

$$w^1 = c \sinh \Psi \qquad \text{and} \qquad w^0 = c \cosh \Psi, \tag{13}$$

where $\Psi = \Psi(\tau)$. Introducing Eq. (13) into Eqs. (11) and (12), we obtain:

$$\overset{\circ}{\Psi} = \Omega(H(\tau)) + \tau_o \Omega(\delta(\tau)), \tag{14}$$

which coincides with the result found by Baylis and Huschilt [6] for the LL equation.

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Step Force for the LL

After a simple integration, considering the initial conditions, we arrive to:

$$\Psi = \left\{ \begin{array}{lll} 0 & \text{for} & \tau < 0 \\ \Omega\tau + \Omega\tau_o & \text{for} & \tau \geq 0 \end{array} \right\} \tag{15}$$

Therefore, we have two cases:
1° case, $\tau < 0$
The solutions are

$$w^1 = 0 \qquad and \qquad w^0 = c \tag{16}$$

$2\circ$ case, $\tau \geq 0$
The solutions are

$$w^1 = c\sinh\left(\Omega\left(\tau + \tau_0\right)\right) \qquad and \qquad w^0 = c\cosh\left(\Omega\left(\tau + \tau_0\right)\right) \tag{17}$$

Graph of w^1 vs τ for LL, $\Omega = 1$ with a Step Electric Field

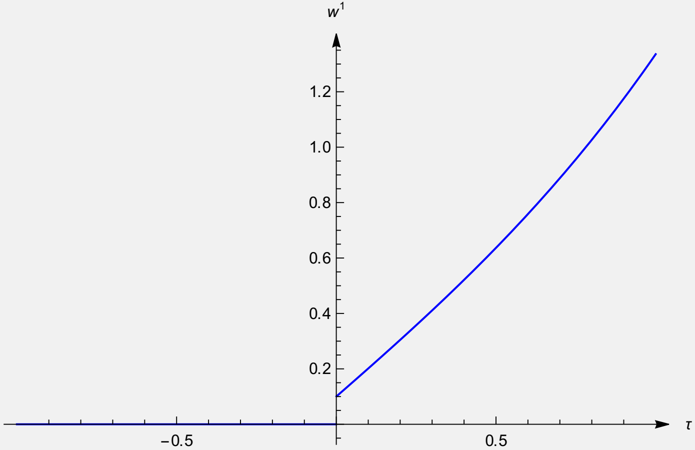

Figure 6: w^1 for the step electric field in the x^1 direction by using the LL

Graph of w^0 vs τ for the LL, $\Omega = 1$ with a Step Electric Field

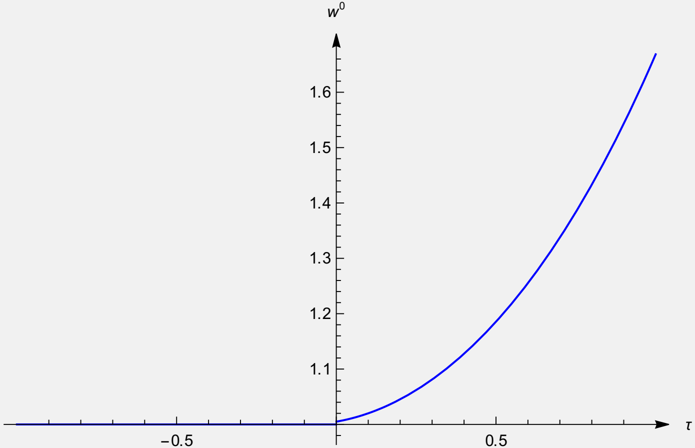

Figure 7: w^0 for the step electric field in the x^1 direction by using the LL

Hammond Theory

The constant force paradox encouraged Hammond to develop a theory which avoids it [3], [7], [8], [9], [10], [11].

He began by proposing an equation of this type

$$\frac{dw^{\mu}}{d\tau} = \frac{e}{mc} F^{\mu\sigma} w_{\sigma} + f^{\mu}, \tag{18}$$

where the radiation reaction force f^{μ} is described by

$$f^{\mu} = \phi^{,\mu} - \frac{w^{\mu}}{c^2} \frac{đ\phi}{d\tau}. \tag{19}$$

It has to be pointed out that đ does not represent an exact differential as it happens with the heat in Thermodynamic. This point represents a correction to Hammond theory. Indeed, we will see that $\phi = \phi(x_{\mu}, w_{\mu})$; that is:

$$\frac{đ\phi}{d\tau} = \frac{\partial \phi}{\partial x_{\mu}} w_{\mu} \qquad and \qquad \frac{d\phi}{d\tau} = \frac{\partial \phi}{\partial x_{\mu}} w_{\mu} + \frac{\partial \phi}{\partial w_{\mu}} a_{\mu}. \tag{20}$$

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Hammond Theory

Physically, this is consistent with the fact that non exact differentials are always connected with no reversible processes as the radiation. Then,

$$a^{\mu} = \frac{dw^{\mu}}{d\tau} = \frac{e}{mc} F^{\mu\sigma} w_{\sigma} + \frac{1}{m} \phi^{,\mu} - \frac{w^{\mu}}{c^2 m} \frac{đ\phi}{d\tau}. \tag{21}$$

Following Hammond [11] but including our correction, we arrive at:

$$P = \frac{đ\phi}{d\tau} = \frac{\partial \phi}{\partial x_{\mu}} w_{\mu} \qquad with \qquad P = -\tau_o m a^2 = -\tau_o m a_{\mu} a^{\mu}. \tag{22}$$

Then,

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Constant Electric Field in the x^1 Direction within Hammond Theory

In order to analyze the constant electric field in the x^1 direction and to be able to solve Eq. (21) it is necessary to make the following approximation (first order in τ_o): The Lorentz acceleration is taken to evaluate the P; that is:

$$\begin{aligned} P &= -\tau_o m a_\mu a^\mu = -\tau_o m \left(\left(\frac{e}{cm} \right)^2 F_{\alpha\nu} w^\nu F^{\alpha\beta} w_\beta \right) \\ &= -\tau_o m \left(\frac{e}{cm} \right)^2 E^2 \left(-w_x w^x - w_0 w^0 \right) \\ &= \tau_o m \frac{e^2}{m^2} E^2 = \tau_o \frac{e^2}{m} E^2. \end{aligned} \tag{24}$$

Therefore,

$$đ\phi = P d\tau = \tau_o \frac{e^2}{m} E^2 d\tau. \tag{25}$$

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Constant Electric Field in the x^1 Direction within Hammond Theory

Knowing that,

$$d\tau = \frac{dt}{\gamma}, \tag{26}$$

we must have

$$\begin{aligned} đ\phi &= \frac{\partial \phi}{\partial x_\mu} dx_\mu = \frac{\partial \phi}{\partial x_0} dx_0 + \frac{\partial \phi}{\partial x_1} dx_1 \\ &= \phi^{,0} dx_0 + \phi^{,1} dx_1 \\ &= P d\tau = P \frac{dt}{\gamma} = \frac{P}{\gamma} dt = \frac{P}{\gamma c} d\left(ct\right) = \frac{P}{\gamma c} dx_0. \end{aligned} \tag{27}$$

Constant Electric Field in the x^1 Direction within Hammond Theory

By using Eqs. (23) y (27), we have

$$\frac{d\phi}{d\tau} = P = \tau_o \frac{e^2}{m} E^2, \qquad \phi^{,0} = \frac{P}{\gamma c} \quad and \quad \phi^{,1} = 0. \tag{28}$$

On the other hand,

$$\begin{aligned} w_\mu \left(\phi^{,\mu} - \frac{w^\mu}{c^2} \frac{đ\phi}{d\tau} \right) &= w_\mu \phi^{,\mu} - \frac{w_\mu w^\mu}{c^2} \frac{đ\phi}{d\tau} \\ &= w_\mu \frac{\partial \phi}{\partial x_\mu} - w_\mu \frac{\partial \phi}{\partial x_\mu} = 0. \end{aligned} \tag{29}$$

This result used in Eq. (21) permits to check the balance of energy. It has to be remembered that in general $\phi = \phi(x^\mu, w^\mu)$. However,

$$\phi^{,0} = \frac{\partial \phi}{\partial x_0} = \frac{P}{\gamma c} = \tau_o \frac{e^2}{\gamma c m} E^2 = \tau_o \frac{e^2}{\gamma c m} E^2, \tag{30}$$

Constant Electric Field in the x^1 Direction within Hammond Theory

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Constant Electric Field in the x^1 Direction within Hammond Theory

Had we used $d\phi$, we will have

$$\begin{aligned} w_\mu\left(\phi^{,\mu}-\frac{w^\mu}{c^2}\frac{d\phi}{d\tau}\right) &= w_\mu\phi^{,\mu}-\frac{w_\mu w^\mu}{c^2}\frac{d\phi}{d\tau} \\ &= w_\mu\phi^{,\mu}-\frac{d\phi}{d\tau} \\ &= w_\mu\phi^{,\mu}-w_\mu\phi^{,\mu}-a_\mu\frac{\partial\phi}{\partial w_\mu} \\ &= -a_\mu\frac{\partial\phi}{\partial w_\mu}\neq 0. \end{aligned} \tag{31}$$

If we analyze Eq. (30), we can notice that $\phi=\phi(w_0)$ since $\gamma c = w_0$. Moreover, Eq. (29) will not be accomplished and the balance of energy will be not satisfied. Therefore, we must use đϕ.

Finally, the radiation reaction term depends on the trajectory as it is expected.

Constant Electric Field in the x^1 Direction within Hammond Theory

Had we used $d\phi$, we will have

$$\begin{aligned} w_\mu \left(\phi^{,\mu} - \frac{w^\mu}{c^2}\frac{d\phi}{d\tau} \right) &= w_\mu \phi^{,\mu} - \frac{w_\mu w^\mu}{c^2}\frac{d\phi}{d\tau} \\ &= w_\mu \phi^{,\mu} - \frac{d\phi}{d\tau} \\ &= w_\mu \phi^{,\mu} - w_\mu \phi^{,\mu} - a_\mu \frac{\partial \phi}{\partial w_\mu} \\ &= -a_\mu \frac{\partial \phi}{\partial w_\mu} \neq 0. \end{aligned} \tag{31}$$

If we analyze Eq. (30), we can notice that $\phi = \phi(w_0)$ since $\gamma c = w_0$. Moreover, Eq. (29) will not be accomplished and the balance of energy will be not satisfied. Therefore, we must use $đ\phi$.
Finally, the radiation reaction term depends on the trajectory as it is expected.

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

We are able to express the equations of motion in such a case

$$\frac{dw^0}{d\tau} = \frac{eE}{cm}w^x + \frac{1}{m}\frac{P}{\gamma c} - \frac{w^0}{c^2 m}P$$

$$\frac{dw^0}{d\tau} = \frac{eE}{mc}w^x + \frac{1}{m}\frac{P}{\gamma c} - \frac{1}{m}\frac{P}{c}\gamma. \tag{32}$$

We obtain

$$c^2 m\dot{\gamma} = eE + \frac{P}{\gamma} - P\gamma. \tag{33}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

We are able to express the equations of motion in such a case

$$\frac{dw^0}{d\tau} = \frac{eE}{cm} w^x + \frac{1}{m}\frac{P}{\gamma c} - \frac{w^0}{c^2 m} P$$

$$\frac{dw^0}{d\tau} = \frac{eE}{mc} w^x + \frac{1}{m}\frac{P}{\gamma c} - \frac{1}{m}\frac{P}{c}\gamma. \tag{32}$$

We obtain

$$c^2 m\dot{\gamma} = eE + \frac{P}{\gamma} - P\gamma. \tag{33}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

For x^1, by using $\Omega = eE/mc$, we arrive at

$$\begin{aligned} \frac{dw^1}{d\tau} &= \frac{e}{mc}F^{\mu\sigma}w_\sigma + \frac{1}{m}\phi^{,1} - \frac{1}{m}\frac{w^1}{c^2}\frac{\text{đ}\phi}{d\tau} \\ \frac{dw^1}{d\tau} &= \frac{eE}{mc}w^0 - \tau_o w^1 \frac{e^2}{c^2m^2}E^2. \end{aligned} \tag{34}$$

That is,

$$\frac{dw^1}{d\tau} = \Omega w^0 - \tau_o \Omega^2 w^1. \tag{35}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Let us propose

$$w^\mu = u^\mu + \tau_o v^\mu. \tag{36}$$

Therefore, Eq. (35) can be written as

$$\frac{dw^1}{d\tau} = \frac{d\left(u^1 + \tau_o v^1\right)}{d\tau} = \Omega\left(u^0 + \tau_o v^0\right) - \tau_o \Omega^2 \left(u^1 + \tau_o v^1\right). \tag{37}$$

On the other hand, developing the identity $w_\mu w^\mu = 1$,

$$\begin{aligned} 1 &= w_0 w^0 + w_1 w^1 \\ &= \left(u_0 + \tau_0 v_0\right)\left(u^0 + \tau_0 v^0\right) + \left(u_1 + \tau_0 v_1\right)\left(u^1 + \tau_0 v^1\right), \\ &= u_0 u^0 + u_1 u^1 + 2\tau_0\left[u_0 v^0 + u_1 v^1\right] + \tau_0^2\left(v_0 v^0 + v_1 v^1\right), \end{aligned}$$

By comparing the coefficients of τ_0 and τ_0^2, we obtain

$$\begin{aligned} u_0 v^0 + u_1 v^1 &= 0, \\ v_0 v^0 + v_1 v^1 &= 0. \end{aligned} \tag{38}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Let us propose

$$w^{\mu} = u^{\mu} + \tau_o v^{\mu}. \tag{36}$$

Therefore, Eq. (35) can be written as

$$\frac{dw^1}{d\tau} = \frac{d\left(u^1 + \tau_o v^1\right)}{d\tau} = \Omega\left(u^0 + \tau_o v^0\right) - \tau_o \Omega^2\left(u^1 + \tau_o v^1\right). \tag{37}$$

On the other hand, developing the identity $w_{\mu} w^{\mu} = 1$,

$$\begin{aligned} 1 &= w_0 w^0 + w_1 w^1 \\ &= \left(u_0 + \tau_0 v_0\right)\left(u^0 + \tau_0 v^0\right) + \left(u_1 + \tau_0 v_1\right)\left(u^1 + \tau_0 v^1\right), \\ &= u_0 u^0 + u_1 u^1 + 2\tau_0\left[u_0 v^0 + u_1 v^1\right] + \tau_0^2\left(v_0 v^0 + v_1 v^1\right), \end{aligned}$$

By comparing the coefficients of τ_0 and τ_0^2, we obtain

$$\begin{aligned} u_0 v^0 + u_1 v^1 &= 0, \\ v_0 v^0 + v_1 v^1 &= 0. \end{aligned} \tag{38}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Therefore,

$$\begin{aligned} v^1 &= -\frac{u_0 v^0}{u_1} = \frac{u^0 v^0}{u^1}, && (39) \\ (v_0)^2 &= -v_1 v^1 = (v^1)^2. && (40) \end{aligned}$$

Then, from Eq. (35), we have

$$\frac{d\left(u^1 + \tau_o v^1\right)}{d\tau} = \Omega\left(u^0 + \tau_o v^0\right) - \tau_o \Omega^2 \left(u^1 + \tau_o v^1\right). \quad (41)$$

Developing in terms of τ_o, we obtain

$$\frac{du^1}{d\tau} = \Omega u^0 \qquad and \qquad \frac{dv^1}{d\tau} = \Omega v^0 - \Omega^2 u^1. \quad (42)$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Since the electric field is constant in x^1 direction,

$$\frac{du^0}{d\tau} = \Omega u^x \Rightarrow u^0 = c\cosh\Omega\tau \quad and \quad \frac{du^x}{d\tau} = \Omega u^0 \Rightarrow u^1 = c\sinh\Omega\tau. \tag{43}$$

Therefore,

$$\frac{dv^1}{d\tau} - \Omega v^0 = -c\Omega^2 \sinh\Omega\tau. \tag{44}$$

Then, we need to express v^0 in order to solve the last equation. From Eq. (67), we have

$$v^0 = \frac{u^1}{u^0} v^1. \tag{45}$$

Then,

$$\frac{dv^1}{d\tau} - \Omega\frac{u^1}{u^0} v^1 = -c\Omega^2 \sinh\Omega\tau. \tag{46}$$

The Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

By using Eq. (43), we arrive at

$$\frac{dv^1}{d\tau} - \Omega \tanh\left(\Omega\tau\right) v^1 = -c\Omega^2 \sinh \Omega\tau. \tag{47}$$

The solution is

$$v^1 = c\tau_0 \Omega \cosh \Omega\tau \left(\Omega\tau - \ln\left(\frac{1+e^{2\Omega\tau}}{2}\right)\right). \tag{48}$$

The Solution of the Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Finally, the solution for w^1 is:

$$\begin{aligned} w^1 &= c\sinh\Omega\tau + \tau_0 v^1 \\ &= c\sinh\Omega\tau + c\tau_0\Omega\cosh\Omega\tau\left(\Omega\tau - \ln\left(\frac{1+e^{2\Omega\tau}}{2}\right)\right) \end{aligned} \tag{49}$$

Now, we need to obtain w^0. We have:

$$\frac{dw^0}{d\tau} = \frac{e}{cm}Ew^1 + \frac{1}{m}\frac{P}{\gamma c} - \frac{w^0}{c^2 m}P.$$

By substituting P, we obtain

$$\frac{dw^0}{d\tau} = \Omega w^1 + \tau_o\Omega^2\left(\frac{c}{\gamma} - w^0\right). \tag{50}$$

The Solution of the Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Finally, the solution for w^1 is:

$$\begin{aligned} w^1 &= c\sinh\Omega\tau + \tau_0 v^1 \\ &= c\sinh\Omega\tau + c\tau_0\Omega\cosh\Omega\tau\left(\Omega\tau - \ln\left(\frac{1+e^{2\Omega\tau}}{2}\right)\right) \end{aligned} \quad (49)$$

Now, we need to obtain w^0. We have:

$$\frac{dw^0}{d\tau} = \frac{e}{cm}Ew^1 + \frac{1}{m}\frac{P}{\gamma c} - \frac{w^0}{c^2 m}P.$$

By substituting P, we obtain

$$\frac{dw^0}{d\tau} = \Omega w^1 + \tau_o\Omega^2\left(\frac{c}{\gamma} - w^0\right). \quad (50)$$

The Solution of the Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

By using Eq. (36) into Ec. (50), with $w^0 = u^0 + \tau_o v^0$, we have

$$
\begin{aligned}
\frac{dw^0}{d\tau} = \frac{du^0 + \tau_o v^0}{d\tau} &= \Omega\left(u^1 + \tau_o v^1\right) \\
&\quad + \tau_o \Omega^2 \left(\frac{c^2}{u^0 + \tau_o v^0} - u^0 + \tau_o v^0\right) \\
\frac{du^0}{d\tau} + \tau_o \frac{dv^0}{d\tau} &= \Omega u^1 + \tau_o \Omega v^1 + \tau_o \Omega^2 \left(\frac{c^2}{u^0} - u^0\right) \quad (51)
\end{aligned}
$$

Therefore,

$$
\frac{du^0}{d\tau} = \Omega u^1 \qquad and \qquad \frac{dv^0}{d\tau} = \Omega v^1 + \Omega^2 \left(\frac{c^2}{u^0} - u^0\right). \qquad (52)
$$

We can assure that:

$$
u^0 = c \cosh \Omega\tau. \qquad (53)
$$

The Solution of the Equation of Motion for the Constant Electric Field in the x^1 Direction within Hammond Theory

Instead of solving directly Eq. (52), from Eq. (70), $v^0 = \frac{u^1}{u^0} v^1$, we can deduce

$$v^0 = \frac{u^1}{u^0} v^1 = \tanh\left(\Omega\tau\right) v^1 = c\Omega \sinh \Omega\tau \left(\Omega\tau - \ln\left(\frac{1 + e^{2\Omega\tau}}{2}\right)\right) \tag{54}$$

Finally,

$$w^0 = c \cosh \Omega\tau + c\tau_0 \Omega \sinh \Omega\tau \left(\Omega\tau - \ln\left(\frac{1 + e^{2\Omega\tau}}{2}\right)\right). \tag{55}$$

Graph of w^1 vs τ with a Constant Electric Field within Hammond Theory, $\Omega = 1$ and $\tau_0 = 0.1$

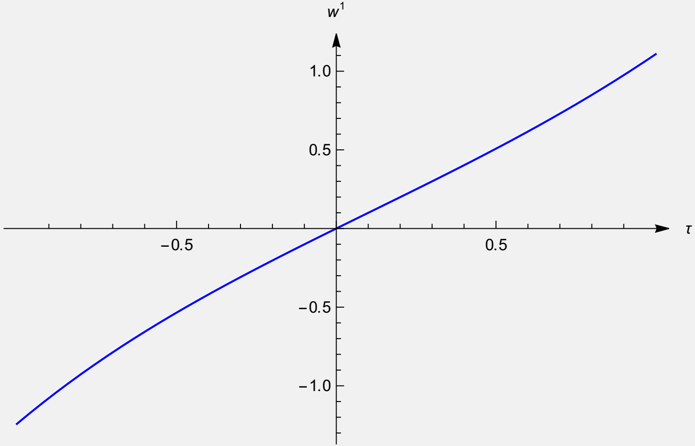

Figure 8: w^1 for the constant electric field in the x^1 direction within Hammond theory.

Graph of w^0 vs τ with a Constant Electric Field within Hammond Theory, $\Omega = 1$ and $\tau_0 = 0.1$

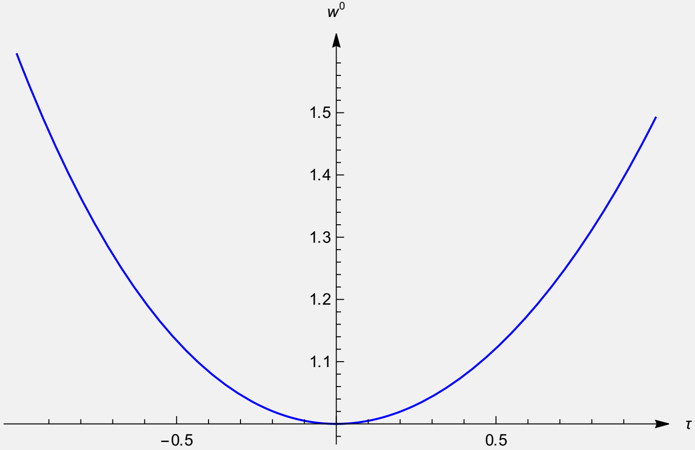

Figure 9: w^0 for the constant electric field in the x^1 direction within Hammond theory.

The Equation of Motion for the Step Electric Field in the x^1 Direction within Hammond Theory

We must now obtain the motion of a charge within Hammond theory in the case of a step electric field in x^1 direction. Therefore, we use the electric field described in Eq. (8). Eq. (47) is still valid

$$\frac{dv^1}{d\tau} - \Omega \tanh\left(\Omega\tau\right) v^1 = -c\Omega^2 \sinh \Omega\tau,$$

but with a different Ω,

$$\Omega = \begin{cases} 0 & for \quad \tau < 0 \\ \frac{eE_o}{cm} = \Omega_0 & for \quad \tau \geq 0 \end{cases} \tag{56}$$

The problem can be divided in two cases:
1° case, $\tau < 0 \Rightarrow \Omega = 0$
Then, Eq. (47) can be written as

$$\frac{dv^1}{d\tau} = 0. \tag{57}$$

The Equation of Motion for the Step Electric Field in the x^1 Direction within Hammond Theory

Then,

$$\frac{dw^1}{d\tau} = 0 \Rightarrow w^1 = 0. \tag{58}$$

2° case, $\tau \geq 0 \Rightarrow \Omega = \frac{eE_o}{cm} = \Omega_0$

First,

$$u^1 = c \sinh \Omega_0 \tau \tag{59}$$

and for v^1

$$\frac{dv^1}{d\tau} - \Omega \tanh(\Omega\tau)\, v^1 = -c\Omega^2 \sinh \Omega\tau, \tag{60}$$

The solution for v^1 is:

$$v^1 = c\tau_0\Omega_0 \cosh \Omega_0\tau \left(\Omega_0\tau - \ln\left(\frac{1+e^{2\Omega_0\tau}}{2}\right)\right). \tag{61}$$

The Equation of Motion for the Step Electric Field in the x^1 Direction within Hammond Theory

Then,

$$\frac{dw^1}{d\tau} = 0 \Rightarrow w^1 = 0. \tag{58}$$

2° case, $\tau \geq 0 \Rightarrow \Omega = \frac{eE_o}{cm} = \Omega_0$

First,

$$u^1 = c \sinh \Omega_0 \tau \tag{59}$$

and for v^1

$$\frac{dv^1}{d\tau} - \Omega \tanh\left(\Omega\tau\right) v^1 = -c\Omega^2 \sinh \Omega\tau, \tag{60}$$

The solution for v^1 is:

$$v^1 = c\tau_0\Omega_0 \cosh \Omega_0\tau \left(\Omega_0\tau - \ln\left(\frac{1 + e^{2\Omega_0\tau}}{2}\right)\right). \tag{61}$$

The Solution of the Equation of Motion with a Step Electric Field in the x^1 Direction within Hammond Theory

Finally,

$$\begin{aligned} w^1 &= c\sinh\Omega_0\tau + \tau_0 v^1 \\ &= c\sinh\Omega_0\tau + c\tau_0\Omega_0\cosh\Omega_0\tau\left(\Omega_0\tau - \ln\left(\frac{1+e^{2\Omega_0\tau}}{2}\right)\right). \end{aligned} \tag{62}$$

Following the same method, we obtain

$$w^0 = c\cosh\Omega_0\tau + c\tau_0\Omega_0\sinh\Omega_0\tau\left(\Omega_0\tau - \ln\left(\frac{1+e^{2\Omega_0\tau}}{2}\right)\right). \tag{63}$$

Graph of w^1 vs τ with a Step Electric Field within Hammond Theory, $\Omega = 1$ and $\tau_0 = 0.1$

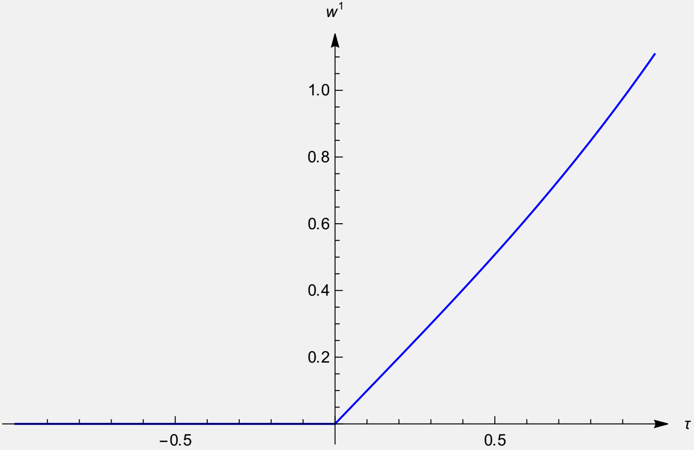

Figure 10: w^1 for step electric field in the x^1 direction within Hammond theory.

Graph of w^0 vs τ with a Step Electric Field within Hammond Theory, $\Omega = 1$ and $\tau_0 = 0.1$

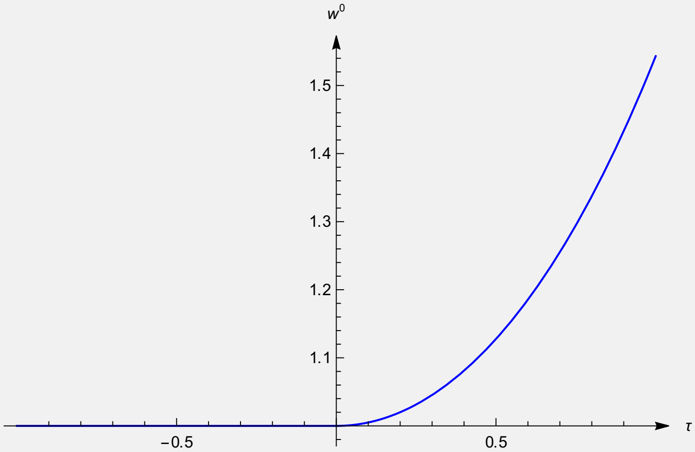

Figure 11: w^0 for step electric field in the x^1 direction within Hammond theory.

Comparison between the Lorentz and LL, and the Hammond Solutions for the Constant Electric field

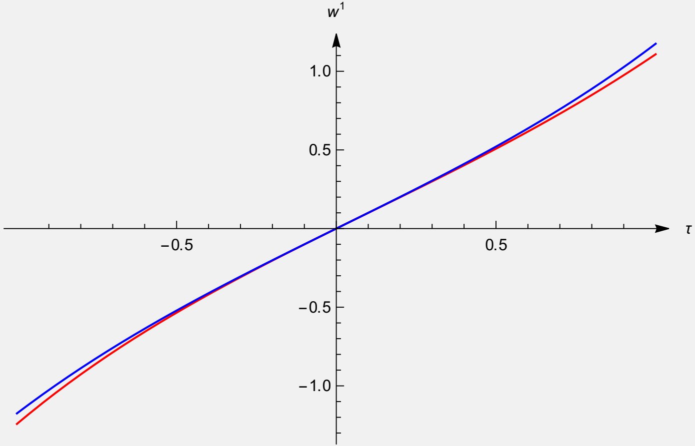

Figure 12: w^1 for Constant Electric Field for Lorentz and LL (in blue) and for Hammond (in red)

Comparison between the Lorentz and the LL, and Hammond Solutions for the Constant Electric field

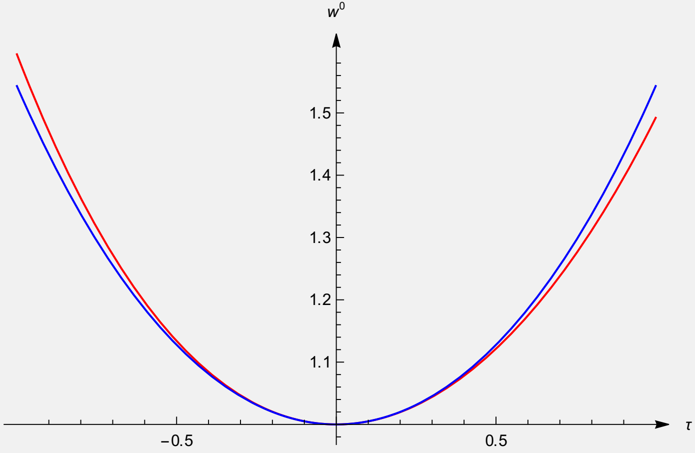

Figure 13: w^0 for constant electric field for Lorentz and LL (in blue) and for Hammond (in red)

Comparison between the Lorentz and the LL, and the Hammond Solutions for the Step Electric field

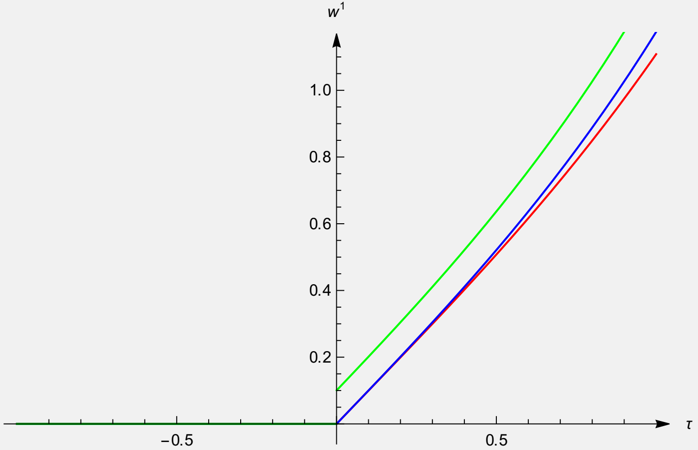

Figure 14: w^1 for step electric field for Lorentz (in blue), LL (in green) and for Hammond (in red)

Comparison between the Lorentz and the LL, and the Hammond Solutions for the Step Electric field

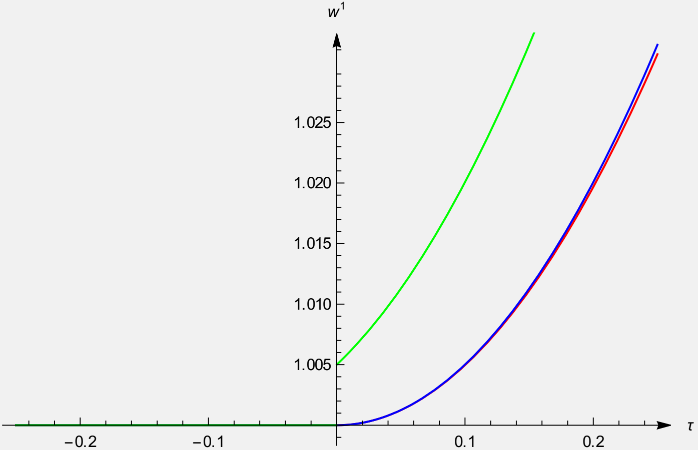

Figure 15: w^0 for step electric field for Lorentz (in blue), LL (in green) and for Hammond (in red)

The Electromagnetic Pulse, Lorentz Case

Let us now consider a polarized electromagnetic pulse, in the x direction ($x = x^1$); that is:

$$\overrightarrow{E} = Eh\,(kz - \omega t)\,\hat{x}, \tag{64}$$

where E is a constant. The corresponding magnetic field is:

$$\overrightarrow{B} = Eh\,(kz - \omega t)\,\hat{y}. \tag{65}$$

By making the following scale transformations $x^\mu \to x^\mu/L$, $t \to tc/L$, $F^{\mu\nu} \to F^{\mu\nu}/E$ with $L = \lambda/2\pi$ and by making the following scale transformations $x^\mu \to x^\mu/L$, $t \to tc/L$, $F^{\mu\nu} \to F^{\mu\nu}/E$ with $L = \lambda/2\pi$, we have

The Electromagnetic Pulse, Lorentz Case

Let us now consider a polarized electromagnetic pulse, in the x direction ($x = x^1$); that is:

$$\overrightarrow{E} = Eh\left(kz - \omega t\right)\hat{x}, \tag{64}$$

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By making the following scale transformations $x^\mu \to x^\mu/L$, $t \to tc/L$, $F^{\mu\nu} \to F^{\mu\nu}/E$ with $L = \lambda/2\pi$ and by making the following scale transformations $x^\mu \to x^\mu/L$, $t \to tc/L$, $F^{\mu\nu} \to F^{\mu\nu}/E$ with $L = \lambda/2\pi$, we have

The Electromagnetic Pulse, Lorentz Case

$$h = \frac{1}{w} e^{-((z-t)/w)^2} \cos\left(\Omega\left(z - t\right)\right) \tag{66}$$

Let us put $a = \frac{eEh}{mc}$, then the Lorentz equation can be written as

$$\begin{aligned} \frac{dw_0}{d\tau} &= ahw_x, && (67) \\ \frac{dw_x}{d\tau} &= ah\left(w_0 - w_z\right), && (68) \\ \frac{dw_y}{d\tau} &= 0, && (69) \\ \frac{dw_z}{d\tau} &= ahw_x. && (70) \end{aligned}$$

The Electromagnetic Pulse, Lorentz Case

From Eqs. (67) and (69)

$$\frac{dw_0}{d\tau} = \frac{dw_z}{d\tau} \tag{71}$$

Integrating $\tau = 0$ a τ

$$w_0(\tau) - w_0(0) = w_z(\tau) - w_z(0),$$

which can be written as

$$w_0(\tau) = 1 + w_z(\tau), \tag{72}$$

By using the initial conditions, $w_0(0) - w_z(0) = 1$, and integrating Eq.(72), we obtain

$$\tau = t - z, \tag{73}$$

which represents an important result.

The Electromagnetic Pulse, Lorentz Case

Then, we can write

$$h = \frac{1}{w} e^{-(\tau/w)^2} \cos(\Omega\tau) \tag{74}$$

where w and Ω represent dimensionless parameters related with the wavenumber and the frequency, respectively.

The solutions are (with the same initials conditions, but including $w^2(0) = w^3(0) = 0$)

$$\begin{aligned} w^0 &= 1 + a^2\mathcal{E}^2, & w^1 &= a\mathcal{E}, \\ w^2 &= 0, & w^3 &= a^2\mathcal{E}^2, \end{aligned} \tag{75}$$

where,

$$\mathcal{E}(\tau) = \int_{-\infty}^{\tau/w} e^{-\zeta^2} \cos(2\Lambda\zeta)\, d\zeta.$$

The Electromagnetic Pulse, Lorentz Case

Then, we can write

$$h = \frac{1}{w} e^{-(\tau/w)^2} \cos\left(\Omega\tau\right) \tag{74}$$

where w and Ω represent dimensionless parameters related with the wavenumber and the frequency, respectively.
The solutions are (with the same initials conditions, but including $w^2(0) = w^3(0) = 0$)

$$\begin{aligned} w^0 &= 1 + a^2\mathcal{E}^2, \qquad & w^1 &= a\mathcal{E}, \\ w^2 &= 0, \qquad & w^3 &= a^2\mathcal{E}^2, \end{aligned} \tag{75}$$

where,

$$\mathcal{E}(\tau) = \int_{-\infty}^{\tau/w} e^{-\zeta^2} \cos\left(2\Lambda\zeta\right) d\zeta.$$

The Electromagnetic Pulse, the LL Case

For the same electric pulse, LL are

$$\begin{aligned}
\frac{dw^0}{d\tau} &= a\left(h+\tau_0\overset{\bullet}{h}\right)w^1+\tau_0 a^2h^2(w^0-w^3)(1-w^0(w^0-w^1)), \\
\frac{dw^1}{d\tau} &= a\left(h+\tau_0\overset{\bullet}{h}\right)(w^o-w^3)-\tau_0 a^2h^2(w^0-w^3)^2w^1, \\
\frac{dw^2}{d\tau} &= -\tau_0 a^2h^2(w^0-w^3)^2w^2, \\
\frac{dw^3}{d\tau} &= a\left(h+\tau_0\overset{\bullet}{h}\right)v^1+\tau_0 a^2h^2(w^0-w^3)(1-w^3(w^0-w^1))
\end{aligned} \tag{76}$$

The Electromagnetic Pulse, Hammond LD Case

For the same pulse, Hammond first use a variation of the Lorentz-Dirac equation [LD] [7] that we will call it the Hammond LD case and consists of using the following equation:

$$\frac{dw^\sigma}{d\tau} = aF^{\sigma\mu}w_\mu + \tau_o\left[\frac{d}{d\tau}\left(aF^{\sigma\mu}w_\mu\right) + \left(\overset{\bullet}{w}^\mu\overset{\bullet}{w}_\mu\right)w^\sigma\right] + \mathcal{O}(\tau_o^2), \tag{77}$$

The Electromagnetic Pulse, Hammond LD Case

For the same pulse, the equations are:

$$\begin{aligned}
\frac{dw^0}{d\tau} &= ahw^1 + \tau_o a^2 \dot{h}\mathcal{E} - \tau_o \frac{a^4 h^2 \mathcal{E}^2}{2}, \\
\frac{dw^1}{d\tau} &= ah(w^0 - w^3) + \tau_o a\dot{h} - \tau_o a^3 h^2 \mathcal{E}, \\
\frac{dw^2}{d\tau} &= 0, \\
\frac{dw^3}{d\tau} &= ahw^1 + \tau_o a^2 \dot{h}\mathcal{E} + \tau_o a^2 h^2 - \tau_o \frac{a^4 h^2 \mathcal{E}^2}{2},
\end{aligned} \tag{78}$$

The next figures are obtained putting $\lambda = 5$, $\Omega = 0.1$, $w = 2\lambda/\Omega$, with an intensity $I = 10^{22}$.

w^0 for the Electromagnetic Pulse for Lorentz Case, LL and Hammond LD

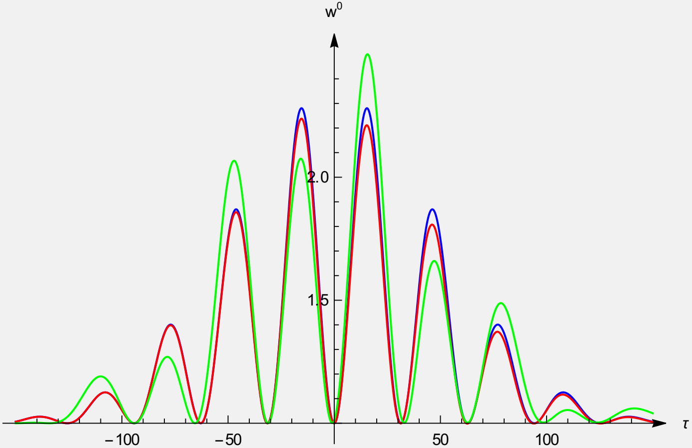


Figure 16: w^0 for the electromagnetic pulse: Lorentz in blue, Hammond LD in red and LL in green

Close-up of w^0 for the Electromagnetic Pulse for Lorentz, LL and Hammond LD

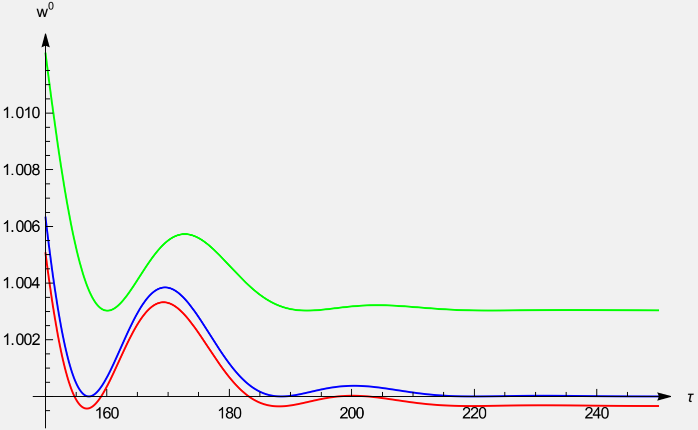
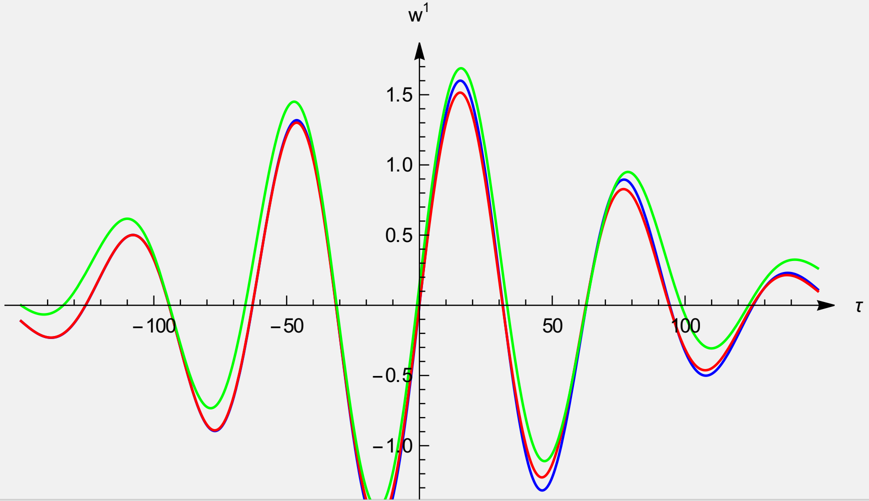


Figure 17: Close-up of the interval (150,250), w^0 for the electromagnetic pulse: Lorentz in blue, Hammond LD in red and LL in green

w^1 for the Electromagnetic Pulse for Lorentz, Landau and Hammond LD



w^3 for the Electromagnetic Pulse for Lorentz, Landau and Hammond LD

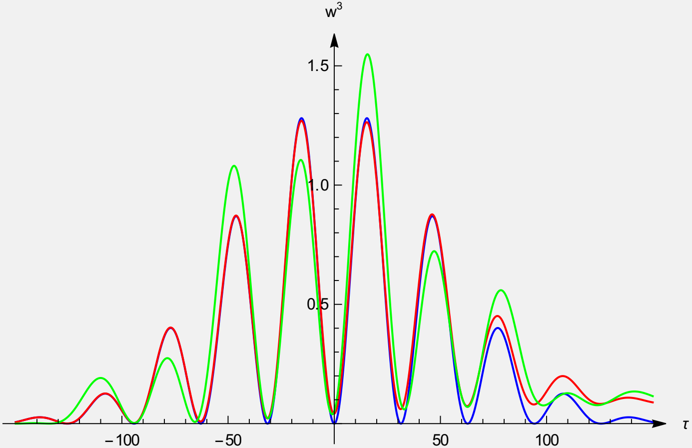


Figure 19: w^3 for the electromagnetic pulse: Lorentz in blue, Hammond LD in red and LL in green, $W^2 = 0$

Close-up of w^3 for the Electromagnetic Pulse: Lorentz in blue, Hammond LD in red and LL in green

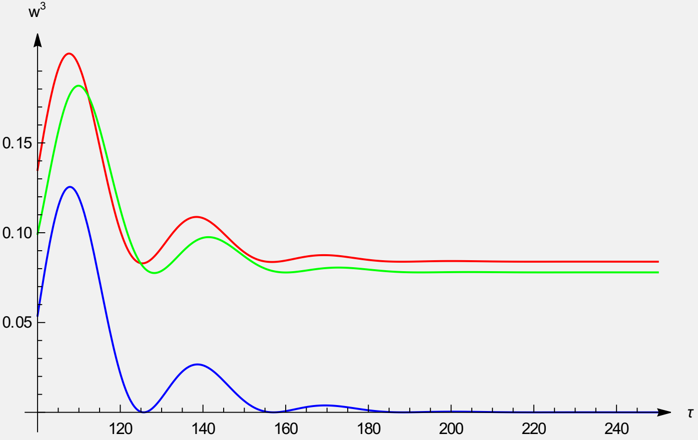


Figure 20: Close-up of the interval (150,250), w^3 for the Electromagnetic pulse: Lorentz in blue, Hammond LD in red and LL in green

The Electromagnetic Pulse, Hammond Approximation Case

Finally, Hammond applied it method for the same pulse but making the following approximation

$$\phi^{,\mu} << \frac{w^{\mu}}{c^2}\frac{\text{đ}\phi}{d\tau}. \tag{79}$$

This is similar with the assumption done by Shen [14] for the Lorentz Dirac equation by neglecting the Schott term. Neverthelees, although he obtained similar results to the ones obtained by using the Hammond LD method, the results obtained for the LL and the Eliezer-Ford-O'Connell [EFO] [12], [13] are completely different.

Graph of the Electromagnetic Pulse, Hammond Approximation in blue, LL in green and EFO in red

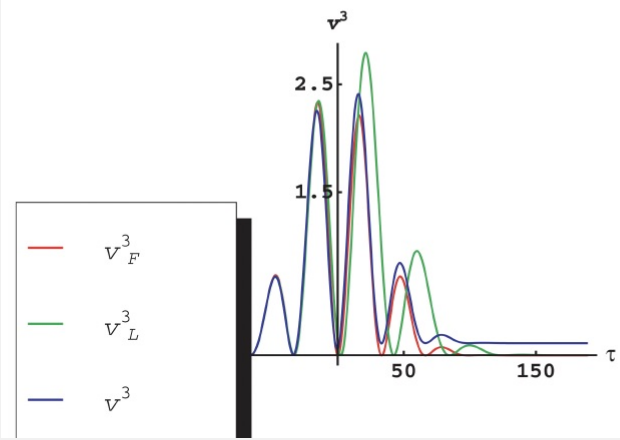


Figure 21: w^3 Hammond Approximation in blue, LL in green and EFO in red

Discussion

Hammond [11] compared the Eliezer-Ford-O'Connell [12], [13], the LL and Hammond results obtaining that for high intense pulses it is possible to experimentally measure the gain of net energy predicted by the Hammond theory described by analyzing the final w^3 components of each case (see figure 2 in [11]).

It has to be noticed that the Lawson-Woodward theorem that states, if radiation reaction is excluded, the particle gains no net momentum from the pulse. This happens when we consider the Lorentz solution.

Also, it has to be highlighted that the LL solutions is very similar to Hammond LD approximation in our solutions.

We notice too that Hammond predicts a different result than ours for the w^3 component which practically coincides with the Hammond LD solutions.

Hammond did not obtain a correct descriptions of the LL and EFO solutions.

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Discussion

For the constant magnetic field, it has been proved that the decay time and trajectories are similar for the Hammond theory [10] and the LL [15]; that is:

$$t_{decay} \propto 1/\tau_o w^2. \tag{80}$$

For the central field case, it has been shown that Hammond theory and LL are equivalent [16].

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Discussion

Finally, we can conclude that the big difference between Hammond theory and LL is the result about the constant force paradox.
However, it is all based on Hammond's ignorance of Schott's energy coming from the field itself generated in the vicinity of the charged particle. In truth, this term causes that in some cases the particle is pushed but always keeping the energy balance.

Discussion

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Discussion

This apparent paradox is explained by other authors by noticing that the radiation exits at the infinity; that is, the energy radiated to infinity is taken from the attached fields (The Schott term or the acceleration energy) and consequently even if the total radiation term in the equation of motion vanishes, the radiation to the infinity (the irreversible emission of radiation) exists. Moreover, by using similar arguments, DeWitt and Brehme explain this phenomenon in his generalization to General Relativity of the damping term [17].

Discussion

Moreover, second, a Landau-Lifshitz-like equation [18] in General Relativity has been proposed supporting the validity of the Landau-Lifshitz equation in Special Relativity.

Finally, and perhaps the most important argument to support the Landau-Lifshitz equation has been done by Krivitskií et al [19] by showing that the radiation reaction term represents an average radiation reaction force in Quantum Electrodynamics.

Due to the nondifferential character of the term $\frac{w^\mu}{c^2}\frac{đ\phi}{d\tau}$, it is impossible to give an equivalent equation of motion for the Hammond theory.

We can conclude that the Landau-Lifshitz equation of motion for a spinless point-like charged particle represents the best proposal to describe the motion of such particles in Classical Electrodynamics.

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End

THANK

YOU

FOR

YOUR

TIME

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